Calculate the ideal chamber pressure for a rocket motor having a K_n of 180

$$K_n = \frac{A_B}{A_*} = 180$$

A_B ← burn area, A_* ← throat area

Steady state chamber pressure = P_0

$$P_0 = \left[\frac{A_B}{A_*} \frac{a \rho_p}{\sqrt{\frac{k}{RT_0}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} \right]^{\frac{1}{1-n}}$$

$$P_0 = \left[180 \cdot \frac{a \rho_p}{\sqrt{\frac{k}{RT_0}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} \right]^{\frac{1}{1-n}}$$

for KNSU, $k = 1.133$

$T_0 = 1720K$

$M = 41.98$ kg/kmol

$R' = 8314$ N-m/kmol K ← universal gas constant

$R = \frac{R'}{M} \Rightarrow R = 198$ Nm/kg-K

wrong / not { by St Robert's law, burn rate can be expressed in terms of chamber pressure

$r = a P_0^n$ { a: burn rate coefficient, n: constant

by chemistry of KNSU:

$a = 0.0665$ in/sec-psi^n ρ_p

$n = 0.319$

$$a = 0.0665 \frac{in}{sec\,psi^n} \cdot \frac{1m}{39.37in} \cdot \frac{1 sec\,psi}{(6895)^{0.319}} = 0.000 \ldots \ Pa^n$$

*Must convert to SI units

Plug in, get:

$P_0 = 4029044$ Pa $= 4.03$ MPa $= 584$ psi

Calculating ideal chamber pressure for APCP Propellant with a K_n of 180-250

(assume 215)

$k = 1.21$ (ratio of specific heats)

$T_0 = 3500°K$ (adiabetic flame temp)

$M = 23.67g/mol$

$R' = 8314$ N-m/kmol-K

$$R = \frac{R'}{M} = \frac{8314.}{25.5g/m} = 326 \text{ N-m/kg-K}$$

$\rho_p = 1680 g/m^{-3}$ (propellant density)

$a = 3.5170541E-5.$ (burn rate coefficient)

$n = 0.3273$ (burn rate exponent)

$$P_0 = \left[\frac{A_b}{A_*} \frac{a\rho_p}{\sqrt{\frac{k}{RT_0}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}}\right]^{\frac{1}{1-n}}$$

Calc one component at a time

$$\frac{A_b}{A_*} = K_n = 215$$

$$\frac{k}{RT_0} = 1.0604773269E-6$$

$$\frac{2}{k+1} = 0.904977375\text{6}$$

$$\frac{k+1}{k-1} = 10.52380952$$

$$\frac{1}{1-n} = 1.486546752$$

$$a\rho_p = 0.0590865089$$

$$P_0 = \left[215 \cdot \frac{0.0590865089}{\sqrt{1.06^{E-6}(0.90)^{10.52}}}\right]^{1.486}$$

12.7

$P_0 = 2635775.616$ Pa

$P_0 = 382$ psi $= 26$ atm

Calculating optimum expansion ratio

$$\frac{A^*}{A_e} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_e}{P_o}\right)^{\frac{1}{k}} \sqrt{\left(\frac{k+1}{k-1}\right)\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]}$$

Solve everything involving involving k

$$\frac{k+1}{2} = 1.105$$

$$\frac{1}{k-1} = 4.7619$$

$$\frac{1}{k} = 0.826$$

$$\frac{k+1}{k-1} = 10.5238$$

$$\frac{k-1}{k} = 0.1735$$

$$\frac{P_e}{P_o} = \frac{1}{26} = 0.0384$$

$$\frac{A^*}{A_e} = (1.105)^{4.7619} (0.0384)^{0.826} \sqrt{(10.52)(1-(0.03)^{0.17}}$$

$$\frac{A^*}{A_e} = 0.232201$$

$$\frac{A_e}{A^*} = 4.30$$

optimal expansion ratio